9. För varje normalriktning $\tilde{n}$ i den i snittytan godtyckligt valda punkten Q existerar en spänningsvektor $\tilde{s}$ definierad av gränsvärdet

$$\tilde{s}(\tilde{n}) = \lim_{\Delta A \to 0} \frac{\Delta R}{\Delta A}$$

$\tilde{s}$ beror alltså på punkten Q:s läge samt riktningen på $\tilde{n}$.

10. $\tilde{e}_x$: utåtriktade normalvektorn i den positiva x-riktningen.

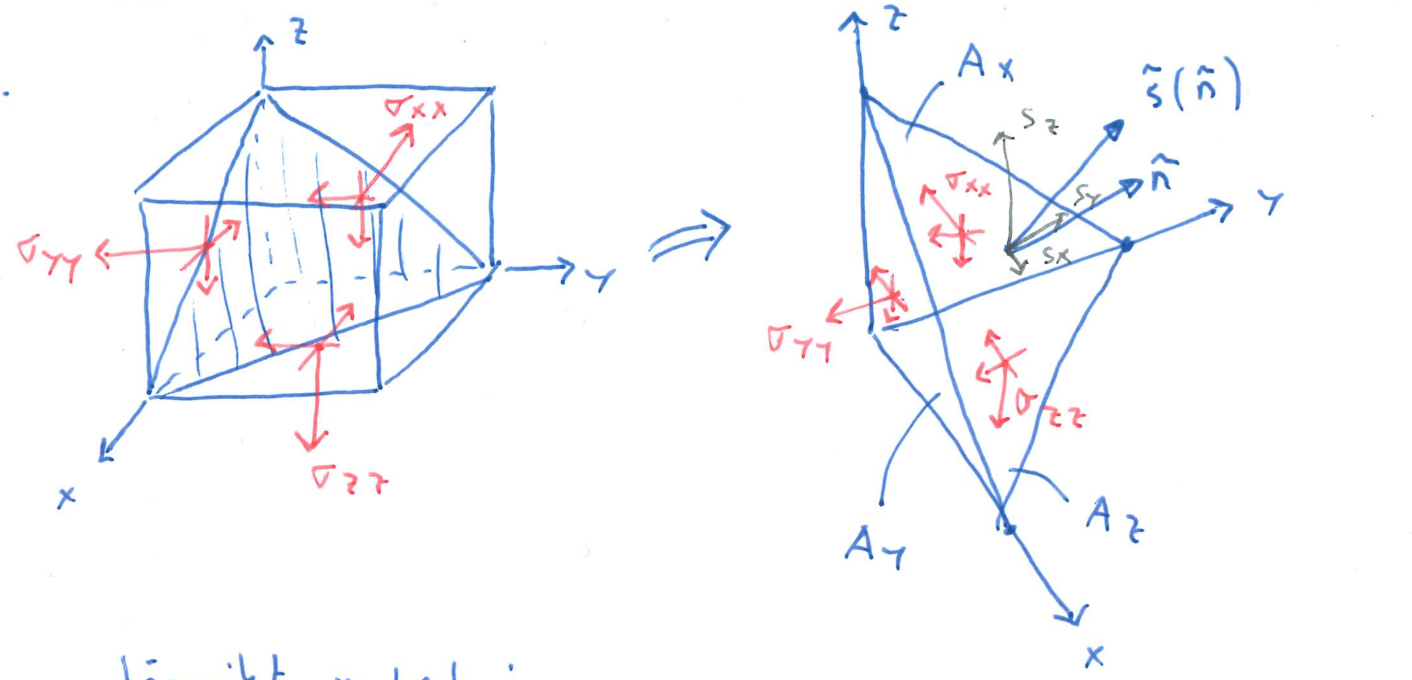
$\sigma_{xx}, \tau_{xy}, \tau_{xz}$. 1:a index anger snittytans normalriktning. 2:a — " — spänningskomponentens riktning.

12. Spänningstillståndet i Q erhålls som gränsvärdet då blocket krymps ihop kring Q $(\Delta x, \Delta y, \Delta z \to 0)$

16. med spänningsmatrisen $\tilde{S}$ känd kan spänningsvektorn på godtycklig snittyta med normal $\tilde{n}$ bestämmas.

A_x, A_y, A_z : sneda ytans projektioner

17.



Jämvikt, x-led:

$$s_x \cdot A - \sigma_{xx} \cdot A_x - \tau_{yx} \cdot A_y - \tau_{zx} \cdot A_z = 0$$

Projicerade areor:

$$A_x = A \cos\alpha = A n_x$$
$$A_y = A \cos\beta = A n_y \quad , \quad \tilde{n} = \begin{pmatrix} n_x \\ n_y \\ n_z \end{pmatrix}$$
$$A_z = A \cos\gamma = A n_z$$

α, β, γ enl. sid. 16

$$s_x = \sigma_{xx} n_x + \tau_{yx} n_y + \tau_{zx} n_z$$

analogt för jämv. i y- och z-led.

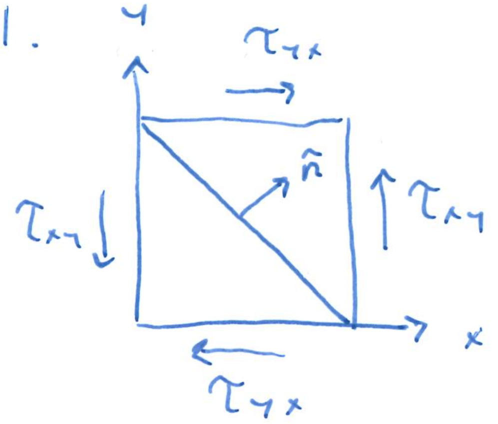
ger:

$$\begin{pmatrix} s_x \\ s_y \\ s_z \end{pmatrix} = \begin{bmatrix} \sigma_x & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \sigma_y & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix} \begin{pmatrix} n_x \\ n_y \\ n_z \end{pmatrix}$$

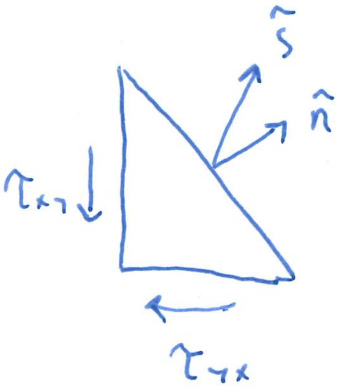
eller

$$\tilde{s} = \tilde{S} : \hat{n}$$, Cauchys formel

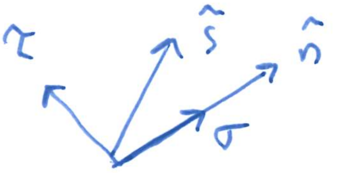
Ex 1.



$$\hat{n} = \begin{pmatrix} \cos 45^\circ \\ \sin 45^\circ \\ 0 \end{pmatrix} = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{pmatrix}$$



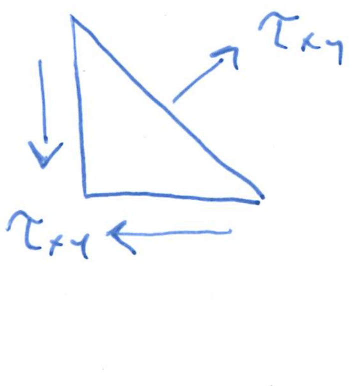
$$\tilde{s} = \tilde{S} \cdot \hat{n} = \begin{bmatrix} 0 & \tau_{xy} & 0 \\ \tau_{xy} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{\tau_{xy}}{\sqrt{2}} \\ \frac{\tau_{xy}}{\sqrt{2}} \\ 0 \end{pmatrix}$$



$$\sigma = \tilde{n}^T \cdot \tilde{s} = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \end{pmatrix} \begin{pmatrix} \frac{\tau_{xy}}{\sqrt{2}} \\ \frac{\tau_{xy}}{\sqrt{2}} \\ 0 \end{pmatrix} = \tau_{xy}$$

$$\tilde{\sigma}_n = \sigma \cdot \hat{n} = \tau_{xy} \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{pmatrix}$$

$$\tilde{\tau} = \tilde{s} - \tilde{\sigma}_n = 0, \quad \text{dvs} \quad \tilde{\sigma}_n = \tilde{s}$$



Ex2. $\tilde{S} = \begin{bmatrix} 0 & \tau_{xy} & 0 \\ \tau_{xy} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Egenvärden ger:

$$\det(\hat{S} - \sigma\hat{I}) = 0$$

$$\begin{vmatrix} -\sigma & \tau_{xy} & 0 \\ \tau_{xy} & -\sigma & 0 \\ 0 & 0 & -\sigma \end{vmatrix} = -\sigma^3 + \sigma\tau_{xy}^2 = 0$$

$$\sigma(\tau_{xy}^2 - \sigma^2) = 0$$

$\sigma = 0$, $\sigma = \pm\tau_{xy}$

$\sigma_1 = \tau_{xy}$, $\sigma_2 = 0$, $\sigma_3 = -\tau_{xy}$

$\sigma_1 > \sigma_2 > \sigma_3$, konvention.

- Huvudspänningsriktningar:

$\sigma_1 = \tau_{xy}$:

$$\begin{bmatrix} -\tau_{xy} & \tau_{xy} & 0 \\ \tau_{xy} & -\tau_{xy} & 0 \\ 0 & 0 & -\tau_{xy} \end{bmatrix} \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix} = 0$$

$$\left.\begin{aligned} -\tau_{xy}\cdot n_1 + \tau_{xy}\cdot n_2 = 0 \quad &\Rightarrow n_1 = n_2 \\ -\tau_{xy}\cdot n_3 = 0 \quad &\Rightarrow n_3 = 0 \end{aligned}\right\} \Rightarrow \hat{n} = \begin{pmatrix} k \\ k \\ 0 \end{pmatrix}$$

$$|\tilde{n}| = 1 \Rightarrow \hat{n} = \frac{1}{\sqrt{k^2+k^2}} \begin{pmatrix} k \\ k \\ 0 \end{pmatrix} = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{pmatrix}$$

$\sigma_2 = 0$:

$$\begin{bmatrix} 0 & \tau_{xy} & 0 \\ \tau_{xy} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix} = 0$$

$$\left.\begin{array}{lll} \tau_{xy} \cdot n_2 = 0 & \Rightarrow & n_2 = 0 \\ \tau_{xy} \cdot n_1 = 0 & \Rightarrow & n_1 = 0 \\ 0 \cdot n_3 = 0 & \Rightarrow & \text{välj } n_3 = 1 \end{array}\right\} \tilde{n} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$\sigma_3 = -\tau_{xy}$:

$$\begin{bmatrix} \tau_{xy} & \tau_{xy} & 0 \\ \tau_{xy} & \tau_{xy} & 0 \\ 0 & 0 & \tau_{xy} \end{bmatrix} \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix} = 0$$

$$\left.\begin{array}{lll} \tau_{xy} \cdot n_1 + \tau_{xy} \cdot n_2 = 0 & \Rightarrow & n_1 = -n_2 \\ \tau_{xy} \cdot n_3 = 0 & \Rightarrow & n_3 = 0 \end{array}\right\} \hat{n} = \begin{pmatrix} k \\ -k \\ 0 \end{pmatrix}$$

$|\tilde{n}| = 1$ ger $\tilde{n} = \begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \\ 0 \end{pmatrix}$